Interstellar Mission

Guide & Answer Key

<u>A Letter to Parents & Teachers:</u>

Hello!

My name is Ari Nathanson. I am a high school student with a passion for both math and theater. In creating this activity, I have sought to bridge these two interests of mine, while also providing parents with an educational tool to use with students who are stuck at home amid the COVID-19 pandemic. Teachers and tutors are also welcome to use or adapt it for math lessons.

This activity can be thought of as a six-act play, with parent/teacher and student as both the actors and the directors. The play tells the story of an interstellar mission in which an experienced rocket captain (played by the student) sets out on a mission to capture an adorable alien to be his boss's daughter's pet. You (the parent/teacher) will play the parts of several characters throughout the captain's journey: the boss, the rocket engineer, the first mate, and the alien.

Throughout the play, your student will be faced with several challenges that require mathematical problem-solving to overcome. These math problems aim to reinforce your student's mathematical knowledge (covering topics including unit conversions and order of operations) and also introduce your student to new, challenging, and exciting areas of math (with more obscure topics like the Chinese Remainder Theorem). **The difficulty of math involved is most appropriate for fifth- through eighth-grade students, although the activity can certainly be used with an older or younger child at your discretion.**

This activity will also engage your student theatrically. Together, you will devise staging for each scene, and also work on building the Captain's character through his lines. You will also get to be creative with your use of props, and you can even design a set if you would like. It might be fun for you and your student to put on a final "performance" of the play to family or friends (after having gone through it once and working through all of the math problems).

The play is composed of six acts. The inner four chapters, which each take around 20 to 30 minutes, contain math problems and riddles built into the story that your child must solve along the way. The prologue and epilogue, each less than 10 minutes long, are math-free scenes that bookend the story. In addition to the performance itself, you will need to set aside time to stage each scene beforehand (and, if you'd like, memorize your lines, although this is definitely not necessary!) I would recommend spacing out these six chapters throughout the course of a week or longer; they are not meant to all be completed in a single day.

Following this letter you will find step-by-step instructions for how you should go about conducting this activity with your student. Beyond that, I have attached the solutions to each of the math problems contained within the script. If you have any questions about the instructions or the solutions, please feel free to contact me by email at **ajn221728@gmail.com** and I will be happy to help.

Thank you for choosing this activity to give your student a fun, unique learning experience. I hope this play sparks a passion for math and theater in your student, and I hope that you have fun along the way.

Good luck, and enjoy!
Ari Nathanson

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Step-by-Step Instructions for Parent and Student:

1. Print out two copies of the script: one for the parent/teacher and one for the student.

2. Print out the answer keys to each problem. The parent/teacher should read over them in advance.

3. For each scene:

    a. Read through the scene once. Look over the math problems but do not attempt to solve them.

    b. "Stage" the scene: find a suitable location for performance, come up with stage directions, work on the delivery of specific lines, etc.

    c. Create a set and costumes. I encourage you to have fun with this and be creative. A cardboard box could easily function as a rocket, and a blanket could serve as a net.

    d. Gather props. Some props, like the deck of cards, are necessary for the math problems; others are optional.

    e. Make sure you have scratch paper, pencils, a calculator, and whatever else you will need to solve the math problems.

    f. Perform the scene, solving the math problems along the way. The parent/teacher can give hints to the student, but ideally should not show the student the answer key. The scene should come to a halt at each problem (it is OK if the actors break character) and should not continue until the student has given the correct solution to the problem.

4. After each scene, take a break. Don't do the entire play in one day. When you are ready, continue with the next scene.

5. When you have finished every scene, you can choose to stage a final performance of the play to friends or family. Instead of solving the math problems along the way, the student can explain his/her solutions to the audience. Being able to explain a mathematical concept to others means you have fully mastered it.

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Chapter 1 Answer Key:

1.1 S-01UT10n is 100,000 light years away. The rocket needs to get to S-01UT10n and back, so we must double this distance to get the total length of the path travelled by the rocket (P).

P = (100,000 ly)(2) = 200,000 ly

1 liter of fuel goes for 5 light years, so the volume of fuel needed (V) in liters is equal to the path traveled in light years divided by 5.

V = 200,000 ly (1 L / 5 ly) = **40,000 L**

1.2 The mass of steel used in constructing the rocket (m_s) is equal to 60% of the total mass of the rocket.

m_s = (0.6)(150 tons) = 90 tons

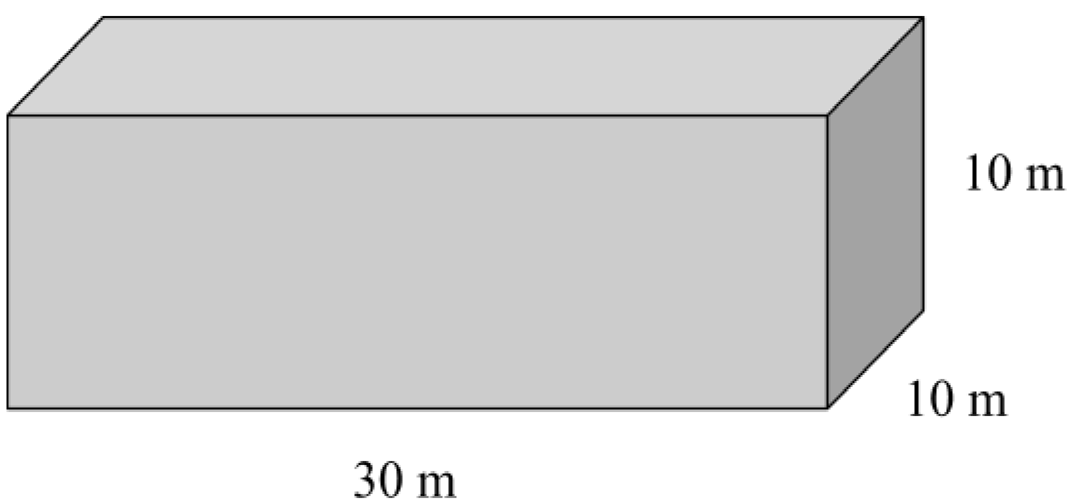
To find the total cost of steel (C_s), we must convert tons to pounds by multiplying by 2,000, and then multiply by $2 per pound.

C_s = 90 tons (2,000 lbs / 1 ton) ($2 / 1 lb) = $360,000

The total cost of the rocket (C) is equal to the total cost of steel plus the total cost of all other materials, which is $100,000.

C = $360,000 + $100,000 = **$460,000** < $500,000

1.3



The first step is to find the total surface area of the rocket. It has six faces, four of which are 30 m x 10 m and two of which are 10 m x 10 m. The surface area (*S*) is equal to the sum of the areas of the six faces, each of which are equal to length times width.

$S = 4\ (30\ \text{m})(10\ \text{m}) + 2\ (10\ \text{m})(10\ \text{m}) = 1{,}200\ \text{m} + 200\ \text{m} = 1{,}400\ \text{m}^2$

The volume of paint needed (*V*) is equal to the surface area times the thickness of the paint coating. To convert this value from cubic meters to liters, we will need to multiply by 1,000.

$V = (1{,}400\ \text{m}^2)(0.002\ \text{m})(1000\ \text{L} / 1\ \text{m}^3) = 2{,}800\ \text{L}$

Each can of paint contains 1 liter, so the answer is **2,800 cans of paint**.

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Chapter 2 Answer Key:

Note: For each of these problems, there may exist solutions beyond those listed. Any solution can be verified using a calculator (or by, but make sure to keep in mind order of operations.

2.1 Some possible solutions:
(5 x 5) + 1 - 2 = 24
1 + (5 x 5) - 2 = 24
(5 x 5) - 2 + 1 = 24

2.2 Some possible solutions:
(12 ÷ 4) + 10 + 11 = 24
11 + (12 ÷ 4) + 10 = 24
10 + (12 ÷ 4) + 11 = 24

2.3 Some possible solutions:
(2 x (9 - 2)) + 10 = 24
10 - ((2 - 9) x 2) = 24
10 + ((9 - 2) x 2) = 24

2.4 Some possible solutions:
(6 ÷ ((5 ÷ 4) - 1)) = 24

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Chapter 3 Answer Key:

3.1 The first step is to figure out how many cards have been removed from the deck (we will call this number N). We know that N is a whole number, and that $1 \leq N \leq 52$. N can be calculated using the following clues:

"If the remaining cards in the deck were dealt to 3 people such that each person were to get an equal number of cards, 1 card would be remaining. If they were dealt to 5 people in this manner, 4 cards would remain. 7 people, and no cards would remain."

Let R represent the number of cards remaining in the deck. We know four things about R:

1. $R = 52 - N$
2. $R = 1 + 3p$, for some whole number p.
3. $R = 4 + 5q$, for some whole number q.
4. $R = 0 + 7r$, for some whole number r.

(Whole numbers are non-negative counting numbers: 0, 1, 2, 3, and so on.)

Based on the first piece of information, that $R = 52 - N$, we know that R is a whole number, and that $0 \leq R \leq 51$.

Based on the second piece of information, we know that R is 1 more than a multiple of 3. This narrows down the possible values of R to 1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, 40, 43, 46, or 49.

Based on the third piece of information, we know that R is 4 more than a multiple of 5. This further narrows down the possible values of R to 4, 19, 34, and 49.

Based on the fourth piece of information, we know that R is a multiple of 7. Out of the four remaining possible values, only 49 is a multiple of 7. Thus, $R = 49$, and $N = 3$.

From here, we can determine the numerical values of the three removed cards using the following clues:

"Out of the cards removed from the deck, no two have the same value. There is one and only one face card, and the rest of the cards have back-to-back values (for example, 2 and 3, or 7, 8, 9, and 10). The values of all the cards add up to 27."

We know that exactly one of the removed cards is a face card, and that the two remaining cards have back-to-back values. Let A represent the value of the smaller card. It follows that the value of the larger card is equal to $A + 1$. Based on the fact that the values of all three cards add up to 27, we have three possible scenarios:

If the face card is a king: $13 + A + (A + 1) = 27 \rightarrow 14 + 2A = 27 \rightarrow 2A = 13 \rightarrow A = 6.5$
If the face card is a queen: $12 + A + (A + 1) = 27 \rightarrow 13 + 2A = 27 \rightarrow 2A = 14 \rightarrow A = 7$
If the face card is a jack: $11 + A + (A + 1) = 27 \rightarrow 12 + 2A = 27 \rightarrow 2A = 15 \rightarrow A = 7.5$

As you can see, both the king and jack scenarios result in a value of A that is not a whole number (this stems from the fact that the sum of two consecutive integers is always odd). A, however, must be a whole number, because all playing cards have whole number values. Thus, the face card must be a queen, meaning that $A = 7$, and the two remaining cards are a seven and an eight.

The suits of the cards can be derived from the following clue:

"The highest-valued card is a heart, the second-highest (if it exists) is a spade, the third-highest (if it exists) a heart, the fourth-highest (if it exists) a spade, and so on."

Therefore, the queen is the **queen of hearts**, the eight is the **eight of spades**, and the seven is the **seven of hearts**.

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Chapter 4 Answer Key:

4.1 *One possible solution is represented below using colored circles (gray represents humans, green represents aliens.) Each of the 11 trips is numbered. To help your student visualize these steps, you may want to use drawings or colored blocks.*

Note: There is at least one other sequence of eleven crossings that is a valid solution to this problem. Any solution can be verified using colored blocks or some other objects to represent the humans and aliens.